Unfolding of reference neutron field spectra at high-energy facilities using the Bonner spectrometer

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Outline

- Motivation
- Measurement of neutron emission spectra with Bonner spectrometers:

  Sensitivity (response) functions
- Regularization methods based on the A.N. Tikhonov approach & TSVD
- Examples of unfolding neutron RF spectra at JINR & CERN
- Conclusion

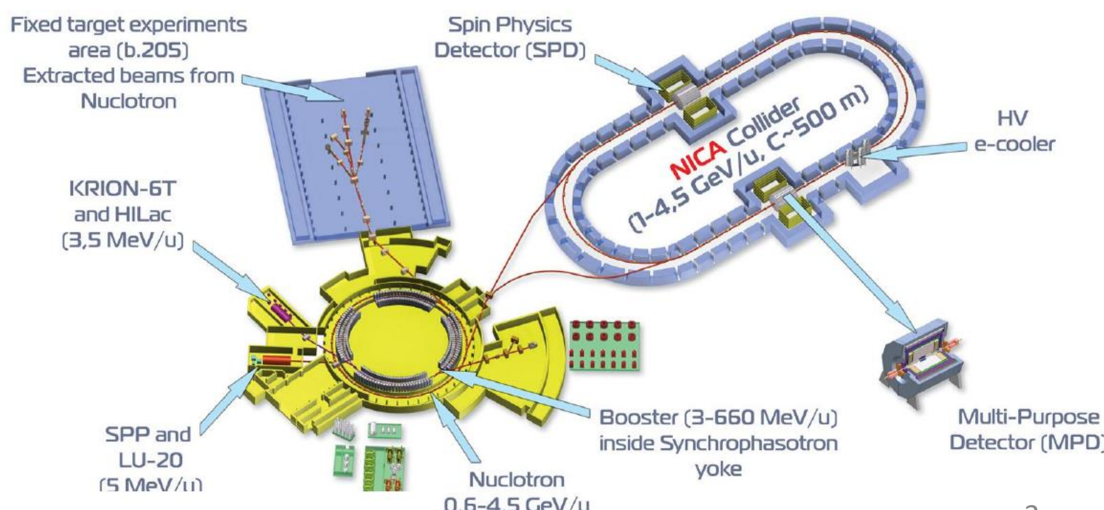
JINR Acceleration Facilities

Phasotron: proton beam 640 MeV

U-400M: heavy ions 50 MeV/u

IREN

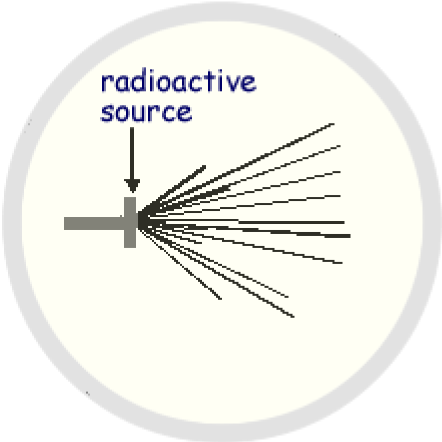
Superconducting accelerator complex NICA
(Nuclotron based Ion Collider fAcility)

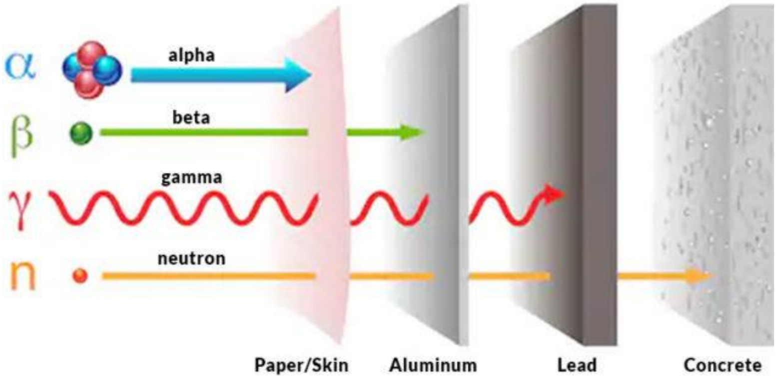


Radiation Protection & Monitoring

Individual dosimetric occupational monitoring is based on the results of dosimetric monitoring of workplaces, taking into account the time a worker spends in these conditions.

- Radiation fields behind the protective shields of nuclear physics facilities (particle accelerator, nuclear reactors) are formed mainly by *neutrons* of a wide energy spectrum because they have *the greatest penetrating power*.
- Radiation monitoring at accelerators (with energies up to several GeV) cannot be carried out using standard dosimeters and neutron radiometers alone, since their operating range is limited by a maximum neutron energy of about 10 MeV.
- *Bonner multi-sphere spectrometer* is a common used tool for radiation monitoring at accelerators.



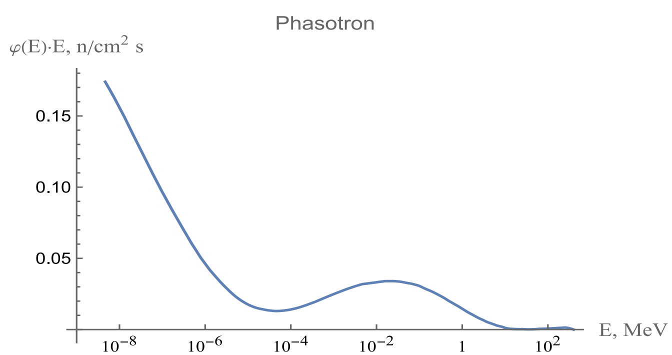


Metrological support for neutron measurements in the high-energy region: Reference fields

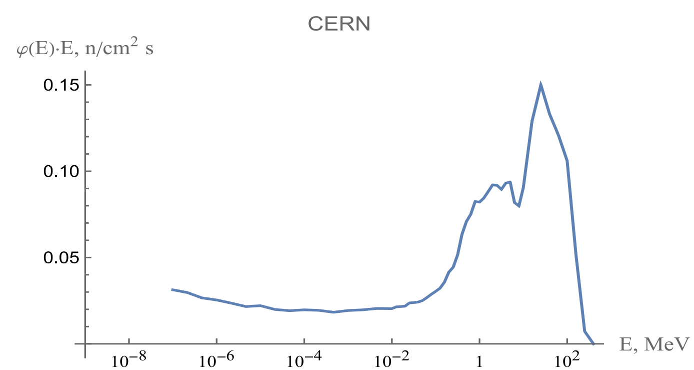
For calibration and verification of dosimeters one uses **standard neutron sources**. However, the miss of standard sources of high-energy neutrons enforces the creation of **reference fields (RF)** at accelerators for the purposes of metrological support of measurements (e.g., for LINUS, WENDI).

RF is understood to be a spatially allocated area of the ionizing radiation field with standardized metrological characteristics.

At JINR: high-energy neutron RF was created behind the side concrete shielding of the Phasotron with a proton energy of 660 MeV.



At CERN: high-energy neutron fields were created behind the concrete overhead shield of the 120 GeV proton beamline.



Note:

Since synchrotrons are essentially **pulsed** radiation sources, **thick** concrete shielding around synchrotrons helps **smooth** out the temporal structure of the field due to differences in neutron transport lengths within an extended volume and thus does not lead to **distortions** in detector readings.

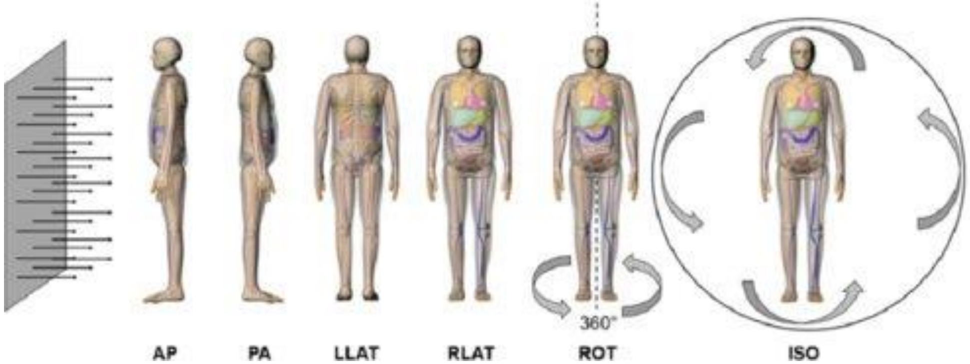
Dose assessment

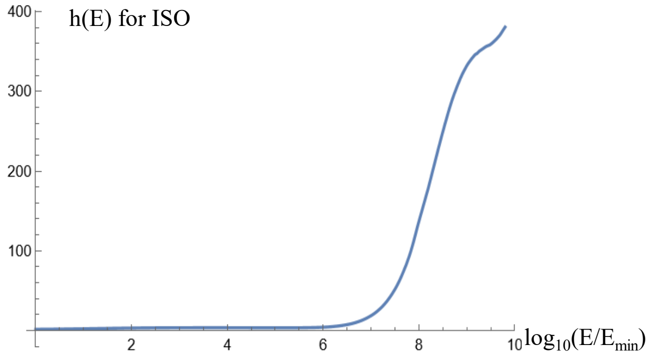
In **RFs**, all necessary studies of *differential and integral characteristics of the field* are carried out and their **reproducibility** is ensured.

One of the most important *differential characteristics* of the neutron field is the *spectral flux density* $\varphi(E)$.
It can be used to assess exposure of personnel in terms of **dose rate**:

$$\dot{H} = \int_{E_{\min}}^{E_{\max}} h(E) \cdot \varphi(E) dE ,$$

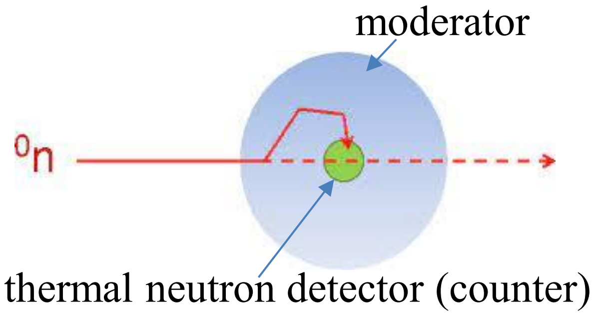
where $h(E)$ [pSv·cm^2] is the **dose conversional coefficient** for mono energetic neutrons in various irradiation geometries for different irradiation types: *AP, PA, LLAT, RLAT, ROT, ISO*

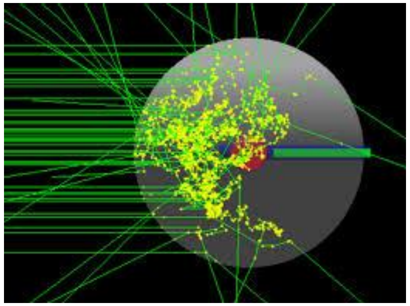


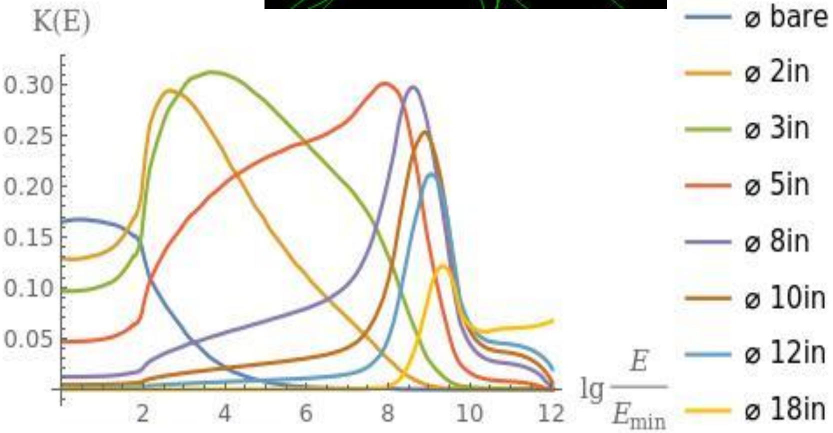


Measurements using Bonner spectrometers

Bonner spectrometer consists of a thermal neutron detector placed in spherical polyethylene moderators of various diameters, the number and size of which are specified by dynamic spectrum range







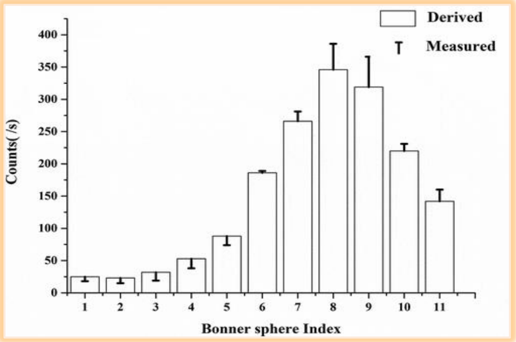
Sensitivity (response) functions for ^{6}Li detector surrounded by 7 moderators

(*Rad. Protect. Dosimetry*, ***10*** (*1985*) *89*)

Measurements using Bonner spectrometers

In order to reconstruct the full spectrum of neutrons $\varphi(E)$ from the results of measurements, it is necessary to solve the system of M Fredholm integral equations of the 1st kind

$$\begin{cases} \int\limits_{E_{\min}}^{E_{\max}} K_1(E)\varphi(E)dE = Q_1 + \Delta Q_1 \equiv Q_1, \\ \vdots \\ \int\limits_{E_{\min}}^{E_{\max}} K_M(E)\varphi(E)dE = Q_M + \Delta Q_M \equiv Q_M \end{cases}$$



where Q_j is the Bonner spectrometer reading for the j-th sphere, and M is the number of spheres used to measure the spectrum, the integration limits $E_{\min}$ and $E_{\max}$ are determined by the spectrum definition area and the set of detectors used for measurements.

In terms of the new variable lethargy $u(E) = \lg(E/E_{\min})$ the integral equations take the form

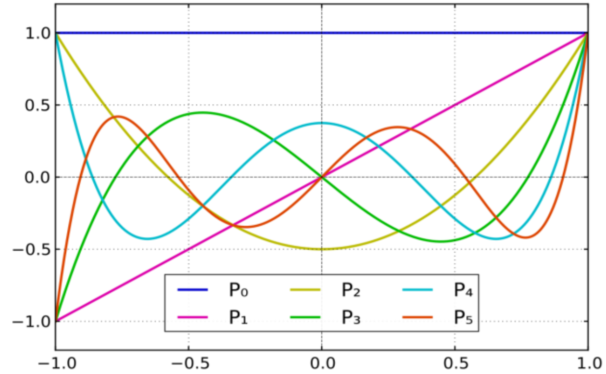
$$\ln 10 \times \int\limits_{0}^{\lg\left(\frac{E_{\max}}{E_{\min}}\right)} K_j(u) \cdot \varphi(u)E(u)du = Q_j, \qquad j = 1, \dots, M.$$

Therefore, instead of the spectrum $\varphi(E)$, it is more rational to find its product $\varphi(u)E(u)$.

Solution by expansion method in sensitivity functions

Our approach to finding the product $\varphi(u)E(u)$is to expand it into *N shifted Legendre polynomials*

$$\Phi(u) \equiv \varphi(u)E(u) = \sum_{j=1}^{N} C_j \cdot P_j\left(\frac{2u}{l_E} - 1\right)$$



As a result, to find the unknown expansion coefficients C_j we obtain a system of *N* linear algebraic equations

$$\boldsymbol{A}^T\mathbf{AC} = \boldsymbol{A}^T\mathbf{Q},$$

where elements of symmetric matrix $\mathbf{A}_{\boldsymbol{M}\times\boldsymbol{N}}$

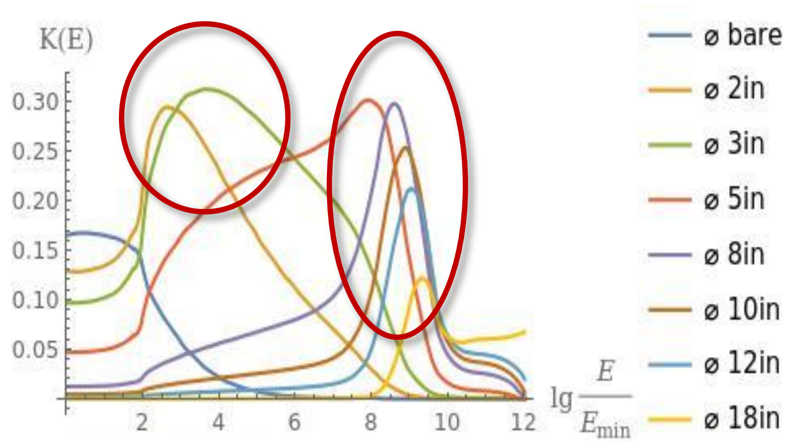
$$A_{ij} = \ln 10 \times \int_0^{l_E} K_i(u) \cdot P_j\left(\frac{2u}{l_E} - 1\right) du\,,$$

and *N*-dimensional vector $\mathbf{C}^T = (C_1, \dots, C_N)$, *M*-dimensional vector $\mathbf{Q}^T = (Q_1, \dots, Q_M)$.

Matrix conditioning

However, a direct solution of such a system of equations by inverting matrix $\boldsymbol{A}^T\mathbf{A}$ does not give a physically justified solution to the spectrum due to the fact that *this matrix is ill-conditioned.*

From a physical point of view, the ill-conditioning of the matrix $\boldsymbol{A}^T\mathbf{A}$ is associated with a strong mutual *overlap* of the spectrometer sensitivity functions



Thus, to solve the problem, *regularization methods* must be applied.

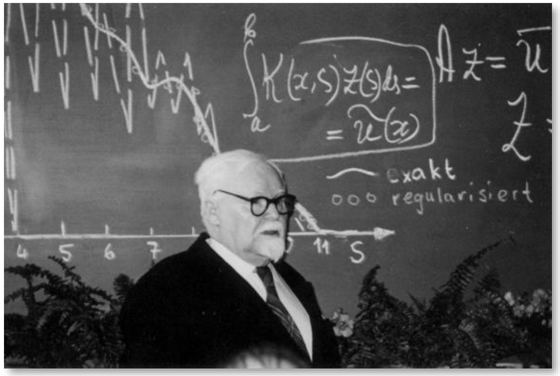
Tikhonov regularization

To apply Tikhonov's regularizing algorithm, we introduce *a stabilizing functional* with mth derivative

$$M^{\alpha}[C] = (\mathbf{AC}-\mathbf{Q})^T(\mathbf{AC}-\mathbf{Q}) + \alpha \times \int_0^{l_E} \left\{\Phi^2(u) + [\Phi'(u)]^2 + \cdots + \left[\Phi^{(m)}(u)\right]^2\right\} du = \sum_{j=1}^{M}\left[\sum_{i=1}^{N} A_{ji}C_i - Q_j\right]^2 + \alpha \times Z,$$

$$Z = \sum_{i,k=1}^{N} C_i C_k \int_0^{l_E} [P_{i-1}(2u/l_E - 1)P_{k-1}(2u/l_E - 1) + P'_{i-1}(2u/l_E - 1)P'_{k-1}(2u/l_E - 1) + \cdots + P_{i-1}^{(m)}(2u/l_E - 1)P_{k-1}^{(m)}(2u/l_E - 1)]du\,,$$

α is *the regularization parameter.*



A.N. Tikhonov
(1906 – 1993)

Tikhonov regularization

From the *condition of the minimum* of this functional

$$\frac{\partial M^{\alpha}[C]}{\partial C_i} = 0,$$

we obtain an algebraic system of linear equations in matrix formwith respect to coefficients $\boldsymbol{C}^{\alpha}$

$$(\boldsymbol{A}^T\boldsymbol{A} + \alpha\boldsymbol{B})\boldsymbol{C}^{\alpha} = \boldsymbol{A}^T\boldsymbol{Q},$$

where

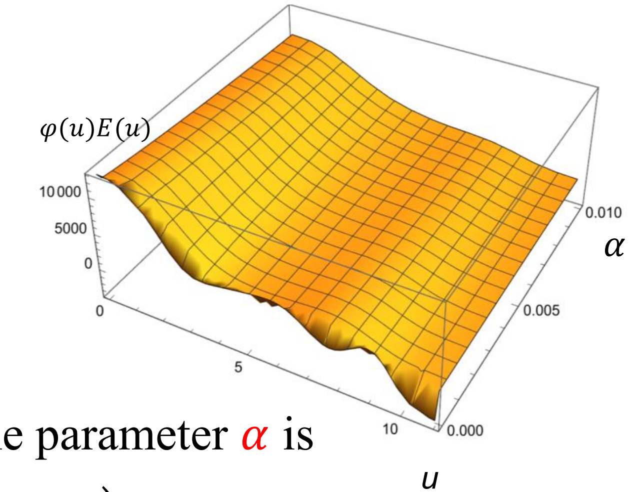
$$B_{ik} = \frac{2l_E}{2i-1}\delta_{ik} + \sum_{n=1}^{\boldsymbol{m}} 4^{1-n}\left(\frac{2}{l_E}\right)^{2n-1} \times \sum_{j=1}^{[N/2]-1} (-1)^{j-1}(n-j)!\,(n+j-1)!\left\{ \binom{i+n-j}{n-j}\binom{i}{n-j}\binom{k+n+j-1}{n+j-1}\binom{k}{n+j-1}\delta_{k-i,2(j-1)} + \binom{k+n-j}{n-j}\binom{k}{n-j}\binom{i+n+j-1}{n+j-1}\binom{i}{n+j-1}\delta_{i-k,2j} \right\}.$$

Tikhonov regularization

The regularization parameter α is found from the equation in accordance with the *generalized residual principle*

$$(\mathbf{A}\boldsymbol{C}^{\alpha} - \mathbf{Q})^{T}(\mathbf{A}\boldsymbol{C}^{\alpha} - \mathbf{Q}) - \delta^{2}\mathbf{Q}^{T}\mathbf{Q} - \mu^{2}(\mathbf{Q}, \mathbf{A}) = 0,$$

where δ is *the relative error* in measurements of the neutron spectrum $\varphi(E)$,
$\mu(\mathbf{Q}, \mathbf{A})$ is a *measure of the incompatibility* of the system of equations taking into account physical restrictions on the spectrum form.



So that the neutron spectrum as a result of such fixation of the parameter α is

$$\Phi^{\alpha}(u) \equiv \varphi^{\alpha}(u)E(u) = \sum_{j=1}^{M} C_j^{\alpha} \cdot P_j\left(\frac{2u}{l_E} - 1\right).$$

Then *the radiation dose rate*

$$\dot{H}^{\alpha} = \int_{E_{\min}}^{E_{\max}} h(E) \cdot \varphi^{\alpha}(E)dE = \ln 10 \times \int_{0}^{\lg\left(\frac{E_{\max}}{E_{\min}}\right)} h(u) \cdot \Phi^{\alpha}(u)du\,,$$

where $h(E)$ are *the dose conversion coefficients* for external radiation.

Truncated SVD method as regularization

Truncated singular value decomposition is a method of *dimensionality reduction* that is able to preserve most of the original information.

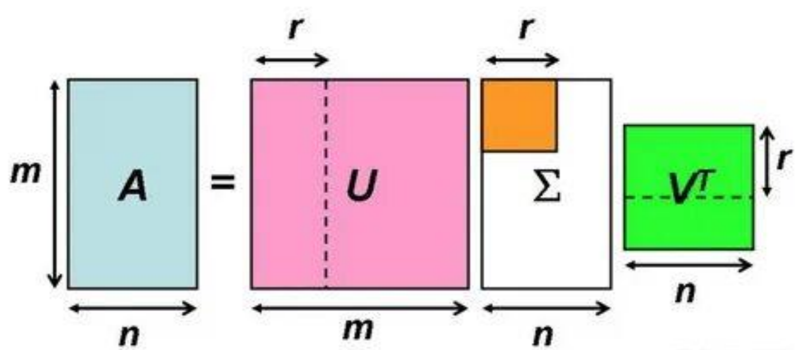
The essence of the method: matrix $\mathbf{A}_{M\times N}$, represented in the **singular value decomposition**

$$\mathbf{A}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T,$$

where unitary matrices $\boldsymbol{U}_{M\times M}, \boldsymbol{V}_{N\times N}$ are *left* and *right singular vectors* and rectangular diagonal matrix $\boldsymbol{\Sigma}_{M\times N} = \mathrm{diag}\{\sigma_1, \sigma_2, \dots, \sigma_M, 0, \dots, 0\}$ with non-negative numbers on the diagonal $\sigma_1 > \sigma_2 > \dots > \sigma_M > 0$ (σ_i being the singular values of $\mathbf{A}$), is approximated by retaining only the r upper singular values and their corresponding singular vectors:

$$\mathbf{A}_r=\mathbf{U}_r\boldsymbol{\Sigma}_r\mathbf{V}_r^T,$$

where $m \times r$ matrix $\mathbf{U}_r$, $k \times r$ matrix $\mathbf{V}_r^T$ and $r \times r$ diagonal matrix $\boldsymbol{\Sigma}_r$.



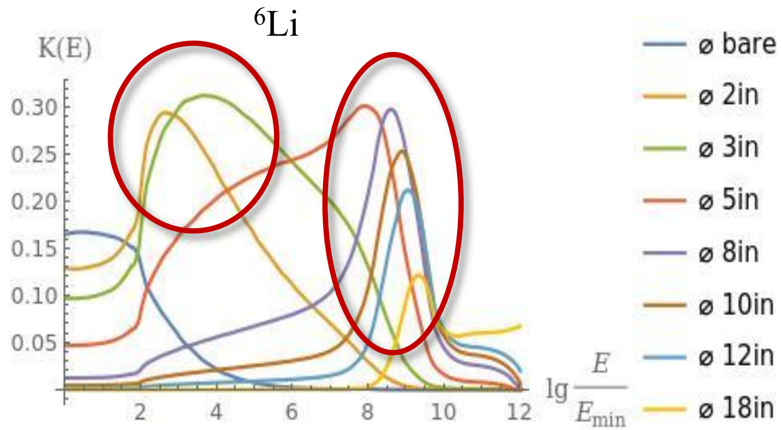
By discarding *small singular values*, the truncated matrix $\mathbf{A}_r$ becomes *more conditioned*:

$$cond(\mathbf{A}_r) = \sigma_1/\sigma_r < \sigma_1/\sigma_M = cond(\mathbf{A}).$$

Therefore, the truncated singular value decomposition can be used as a regularization method for solving a system of equations for the coefficients of the neutron spectrum decomposition.

Trancation vs. sets of moderation spheres

Since the spectrometer sensitivity functions are *not linearly independent*,



the question of choosing the optimal set of spectrometer moderation spheres is rather important
(*Chizhov A., Chizhov K*. Math. Modeling, **8** (2024) 89).

In this regard, a properly chosen truncation parameter r which denotes the actual rank of the matrix in the system of equations can indicate an effective set of moderation spheres M_{eff} for performing measurements of neutron spectra without qualitatively loss in the radiation dose assessment:

$$M_{eff} \sim r.$$

Iterative TSVD method

Thus, to find the expansion coefficients, we solve the following system of N linear algebraic equations with the truncated matrix $\mathbf{A}_r$:

$$\mathbf{A}_r^T\mathbf{A}_r\mathbf{C}_r = \boldsymbol{A}^T\mathbf{Q}.$$

To find a numerical solution to such a system of equations, it is convenient to use *an iterative method*. As such a method, we use the *Landweber algorithm*:

$$\mathbf{C}_r^{(k+1)} = (\mathbf{I} - \tau\mathbf{A}_r^T\mathbf{A}_r)\mathbf{C}_r^{(k)} + \tau\boldsymbol{A}^T\mathbf{Q},$$

where the *relaxation factor* $0 < \tau < 2/\sigma_1^2$ and

$$\mathbf{C}_r^{(0)} = (\mathbf{A}_r^T\mathbf{A}_r)^{-1}\boldsymbol{A}^T\mathbf{Q} = \mathbf{V}_r(\boldsymbol{\Sigma}_r^{-1})^2\mathbf{U}_r^T\boldsymbol{A}^T\mathbf{Q}.$$

So, the neutron spectrum as a result after K iterations has the form

$$\Phi_r^K(u) \equiv \varphi_r^K(u)E(u) = \sum_{j=1}^{N} C_{r,j}^{(K)} \cdot P_j\left(\frac{2u}{l_E} - 1\right).$$

As *the criterion* for choosing the numbers of truncation r and iterations K is *the condition* of ensuring *the non-negativity of* the neutron spectrum $\varphi_r^K(E)$.

Then *the radiation dose rate* can be calculated as

$$\dot{H}_r^K = \int_{E_{\min}}^{E_{\max}} h(E) \cdot \varphi_r^K(E)dE = \ln 10 \times \int_0^{l_E} h(u) \cdot \Phi_r^K(u)du\,,$$

where $h(E)$ are theexternal radiation dose *conversion coefficients*.

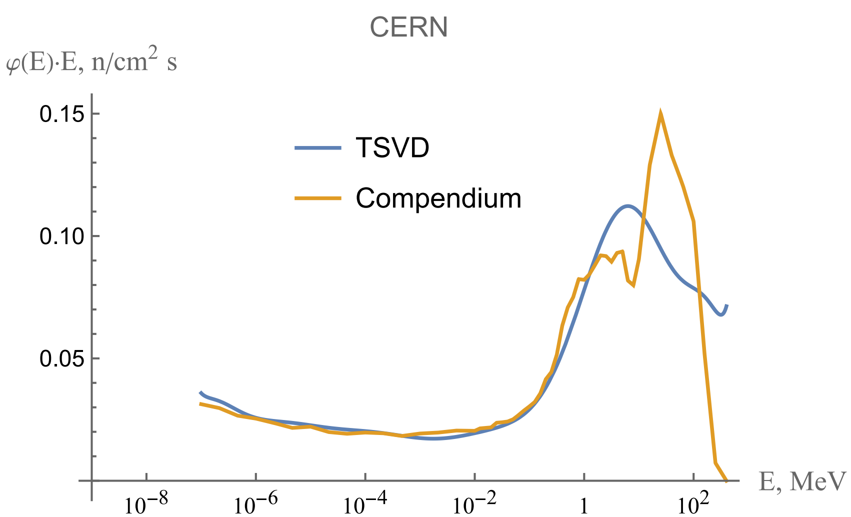
RF spectrum unfolding at CERN: TSVD method

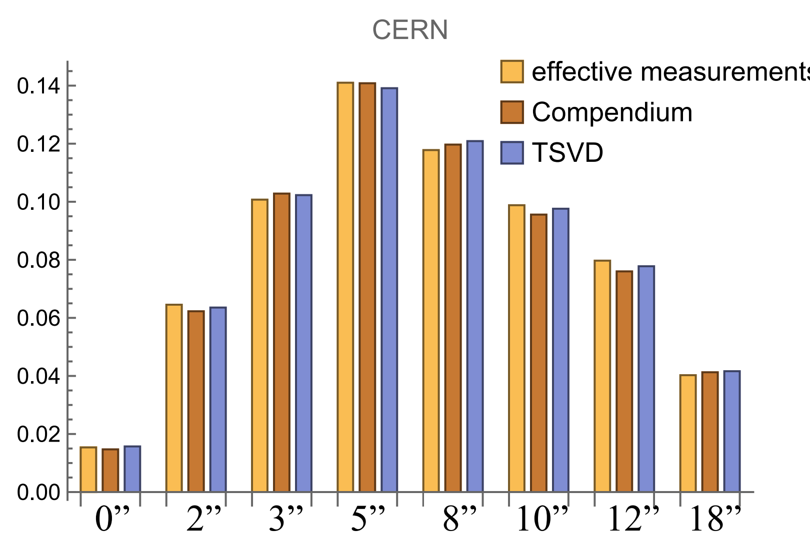
Diameters of $M = 8$ polyethylene spheres = {0”, 2”, 3”, 5”, 8”, 10”, 12”, 18”} surrounded 4 mm x 4 mm ^{6}Li detector (*Awschalom M., Sanna R.S.* Rad. Protect. Dosimetry, **10** (1985) 89).
Spectrometer effective counts:

Q_{eff}= {0.0153802, 0.0645362, 0.100726, 0.140998, 0.117802, 0.0988112, 0.0797087, 0.0402594}
(Compendium); δ_{eff}=0.024 (2.4%)

$E_{\min} = 10^{-7}$ MeV, $E_{\max} = 398$ MeV; $N = 15$ shifted Legendre polynomials: $\{P_0, \dots, P_{14}\}$

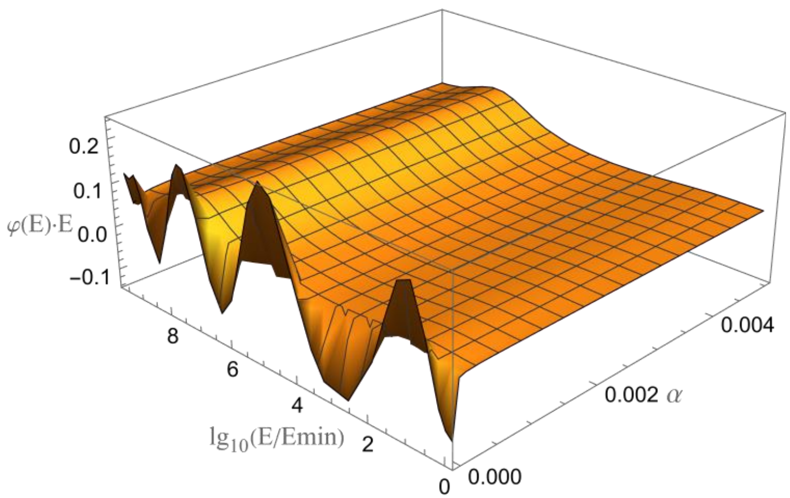
$$\Sigma_r = \mathrm{diag}\{\underbrace{2.95289,\ 0.781667,\ 0.301539,\ 0.131619,\ 0.0680812,\ 0.0256383}_{r=6},\ \underbrace{\cancel{0.00750383,\ 0.000745984}}_{0}\}$$



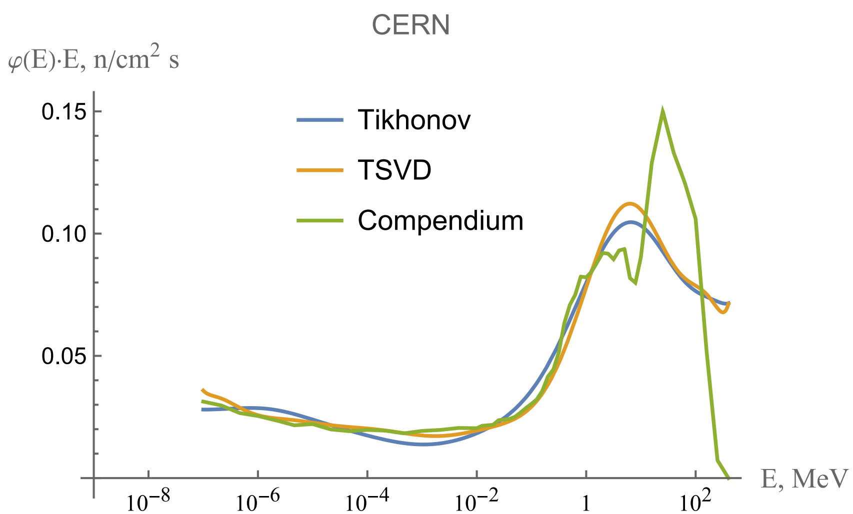


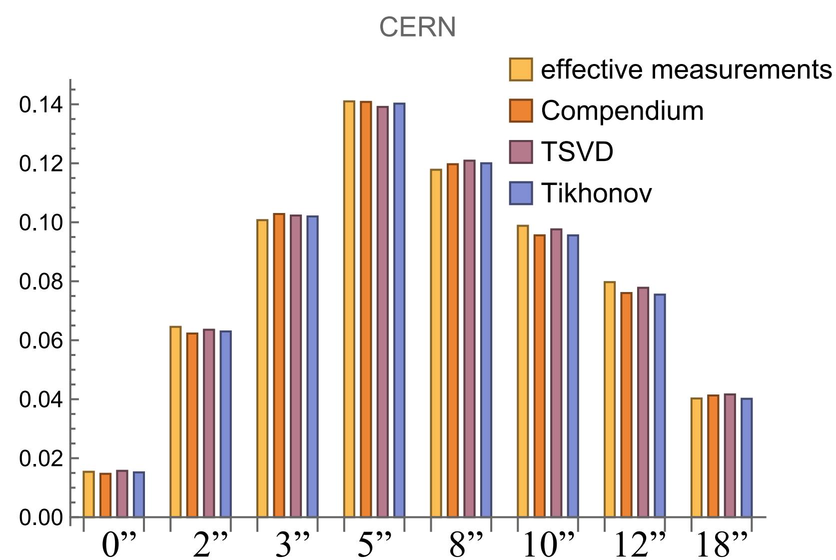
Unfolded RF spectrum by TSVD ($r = 6$) *(left) and effective detector readings (right) with the relative accuracy* $\delta_{eff}^{TSVD} = 0.015$ (1.5%).

RF spectrum unfolding at CERN: Tikhonov method



$\delta_{\mathrm{eff}} = 0.024 \rightarrow \alpha = 0.0027$





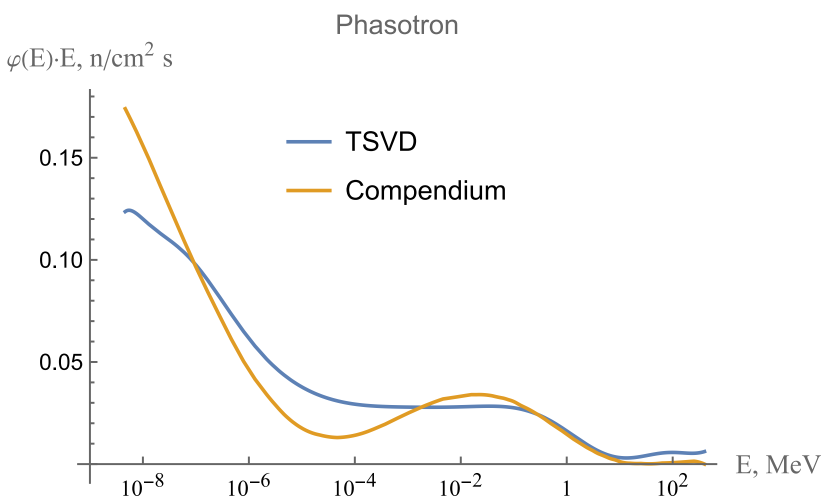
Unfolded RF spectra by Tikhonov regularization and TSVD ($r = 6$) *(left) and effective detector readings (right) with relative accuracies* $\delta_{eff}^{TR} = \delta_{eff} = 0.024$ (2.4%) *and* $\delta_{eff}^{TSVD} = 0.015$ (1.5%).

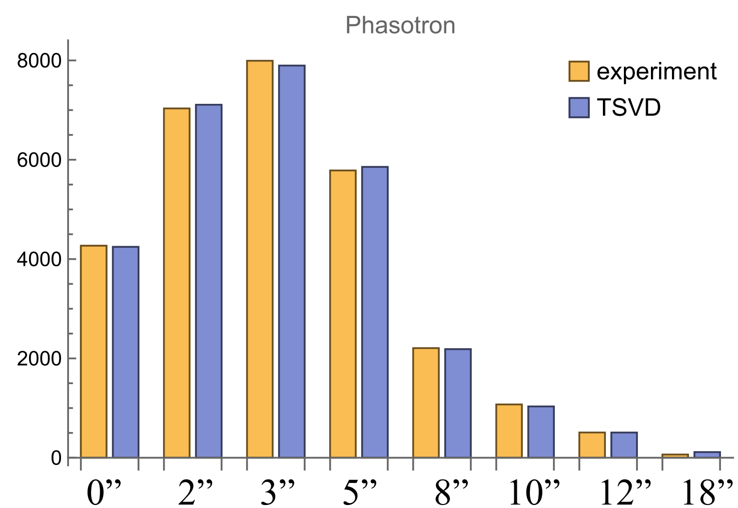
RF spectrum unfolding at Phasotron: TSVD method

Spectrometer counts: $Q_{exp} = \{138.4, 288.9, 404.4, 416.7, 222.7, 137.6, 91.39, 41.82\}$
(*Aleinikov V. et al.* Rad. Protect. Dosimetry, **54** (1994) 57);
$\delta_{exp} = 0.05$ (5%)

$E_{min} = 5 \times 10^{-9}$ MeV, $E_{max} = 398$ MeV; $N = 15$ shifted Legendre polynomials: $\{P_0, \ldots, P_{14}\}$

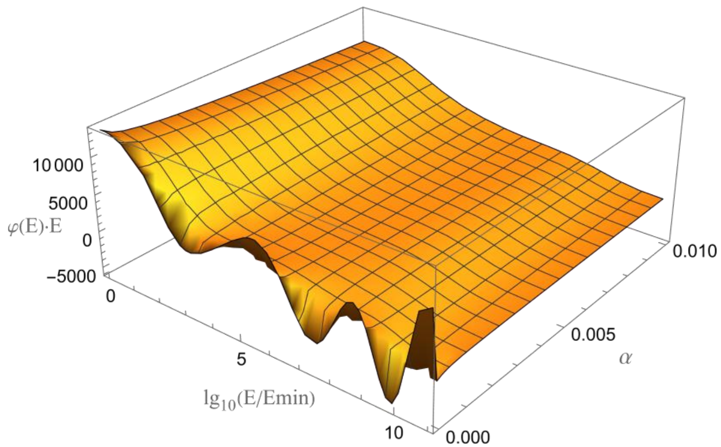
$$\Sigma_r = \mathrm{diag}\{\underbrace{3.17551,\ 0.841275,\ 0.379885, 0.183081, 0.0773665}_{r\,=\,5}, \underbrace{\cancel{0.02807,\ 0.00775113,\ 0.000745156}}_{0}\}$$



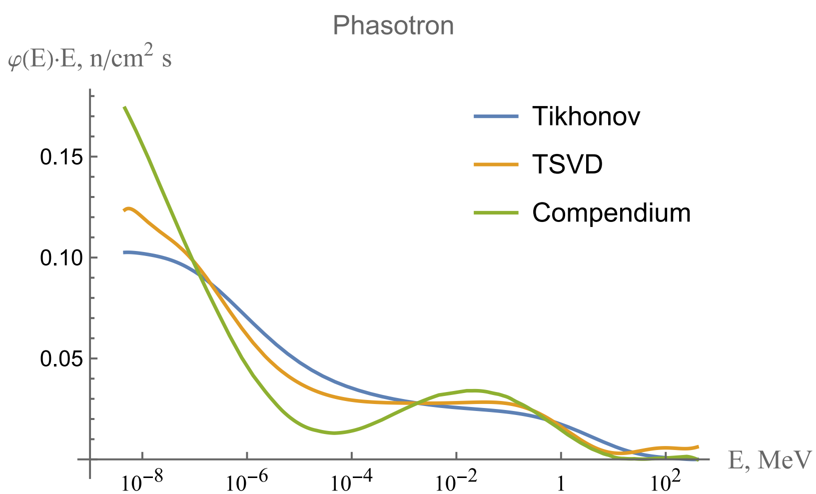


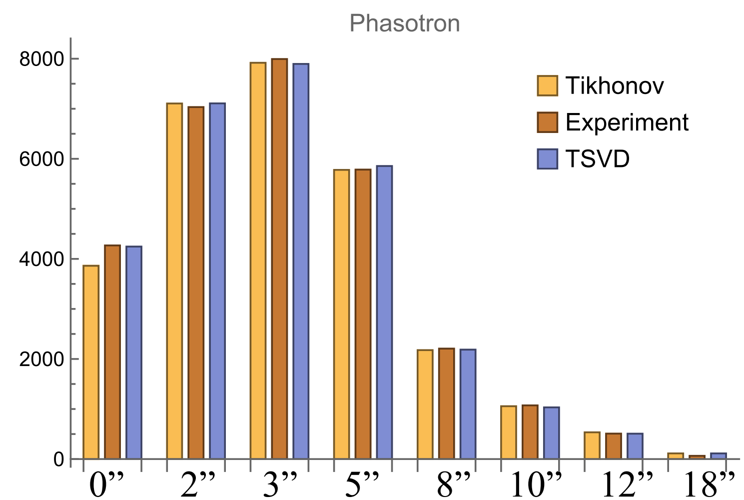
Unfolded RF spectrum by TSVD ($r = 5$) (*left*) *and effective detector readings* (*right*)
with the relative accuracy $\delta_{TSVD} = 0.012$ (1.2%).

RF spectrum unfolding at Phasotron: Tikhonov method



$\delta_{exp} = 0.05 \rightarrow \alpha = 0.0012$





Unfolded RF spectra by Tikhonov regularization and TSVD $(r = 5)$ *(left) and effective detector readings (right) with relative accuracies* $\delta_{TR} = \delta_{exp} = 0.05$ (5%) *and* $\delta_{TSVD} = 0.012$ (1.2%).

Conclusion

- Both methods – Tikhonov and TSVD – allow the unfolding of neutron field spectra in a wide energy range with an accuracy sufficient for the required assessment of personnel radiation doses.
- In addition, TSVD method and the Weighted Tikhonov Regularization we proposed can be used to determine *the optimal set of moderator spheres* (their sizes and number) for effective practical measurements.
- For a more precise resolution of the unfolded spectrum, a more complex configuration of moderator spheres is apparently required.

Publications

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Thank you for your attention!